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A MANOMETER DEVICE FOR AIR THERMOMETERS.

By Harold Edwards.

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Five years since an investigation was begun involving the use of an air thermometer. Before the investigation was completed a publication of the Reichanstalt more than covered the field contemplated. The investigation was, therefore, discontinued. In the meantime an air thermometer had been constructed containing some novel features, one of which in particular is perhaps worth recording, — the method of measuring the height of the mercury column. This is a very old problem in connection with both the air thermometer and the normal barometer, and it is recognized by every one who has worked with either of these instruments as still far from a satisfactory solution. The best methods used have been those suggested by Pernet, Marek, and Thiesen. These depend on the observations of the direct and reflected images of points or lines through the cylindrical walls of heavy glass tubing, and involve errors due to the displacement and distortion of the images by the non-parellelism of the surface, and by the heterogeneity of the glass. Further, the resolving power of the telescope and the brilliancy of the images observed in these methods are diminished greatly when one brings the mercury surface very near to a fixed glass surface, as is necessary at the lower meniscus of a constant-volume gas thermometer. There are besides these many small sources of error familiar to any one who has faced the problem. When the main experiment was abandoned, the work on the air thermometer had been carried far enough to show practically the merits of the device which is here described.

The general arrangement of the apparatus is shown in Figure 1. A, A are two glass plates containing on their inner surfaces the scales on which the mercury height is to be measured. B, B is the mercury column; C an air reservoir at the top of the barometer column, exhausted by an air pump in the usual way. D is the tube leading through the wall to the bulb of the air thermometer in the adjacent room. E, E are the devices for determining the position of the surface of the mercury.

It is to the latter that attention is particularly called in the present note.

In Figure 2 is shown the details of the device E. Each consists of a plate of optically plain parallel glass. Opposite edges of this plate are polished and ruled with a diamond midway between the upper and lower surfaces. Each of these side lines, as they may be called, faces one of the large glass scales. The position of the scales is so adjusted that the distance from plate to scale is as small as possible. On the under side of the plate E and near its centre is a diamond line, perpendicular to the sides turned toward the scales. On the top of the plate E rest two prisms whose position and purpose will be explained later.

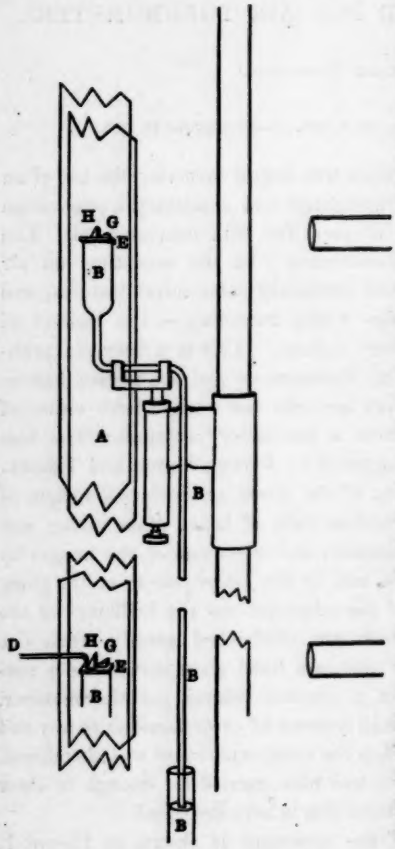

FIGURE 1.

The position of the side lines with reference to the glass scales is read by means of cathetometer telescopes not shown in the drawing.

The distance between each side line and the lower surface of the plate can be determined with great accuracy before the plate is mounted on the apparatus, by observing under a microscope the apparent displacement of the line when the opposite plain surfaces are brought against a fixed stop. Half this displacement is equal to the distance from the line to the central plane of the plate. Knowing the thickness of the plate, the distance from the side lines to the under surface of the plate is determined.

This distance determined, we have left for determination the interval

from the lower surface of the plate to the surface of the mercury. This is accomplished by means of the two glass prisms G and H cemented with Canada balsam on the top of the plate.

In the case of the prism G a beam of light directly from the line is reflected at the surface of the prism and gives an image in the telescope D. Seen in the same prism there will be another image somewhat above the first and coming by reflection from the mercury surface. Measuring the vertical distance between the images by means of a cathetometer telescope we can calculate accurately and easily, if the plate and the telescope are horizontal, the distance between the glass and the mercury surfaces. The prism H gives a pair of images precisely similar, except that the one coming by reflection from the mercury is now below the direct image. Let b and b_1 be the vertical displacement of the telescope for each pair of images,

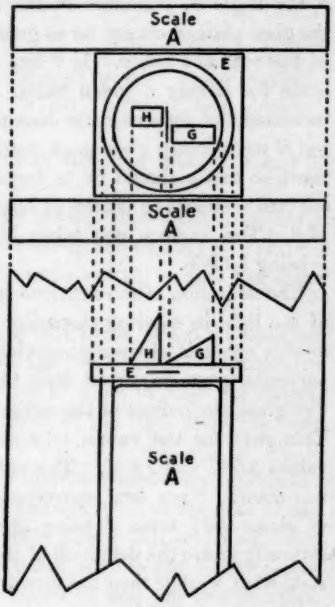

FIGURE 2.

d the distance from the line to the mercury surface, x and x_1 the values of the ratios $\frac{b}{d}$ and $\frac{b_1}{d}$, i. e., the effective magnifying powers of the prisms and plate. The unaccented letters refer to prism G, the accented letters to prism H.

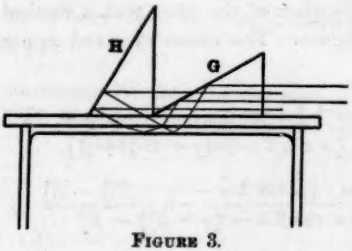

FIGURE 3.

Assuming for the present that the plate and telescope are level, we have, according to Figure 3, α being the angle of the prism and μ the index of refraction of the glass, and the analysis being so simple and obvious as to scarcely need entering here, —

$$x = x' = \frac{b}{d} = \frac{\mu \sin 4\alpha}{\sqrt{1 - \mu^2 \cos^2 2\alpha}}.$$

In selecting the best values for μ and a, the index of refraction of the glass and the angle of the prism, one has to bear in mind that the angle θ, the angle of incidence of the reflected light on the under surface of the glass plate, must not be so great as to cut down seriously the intensity of the reflected beam. If θ has a given value, which is equivalent to $\mu \cos 2a$ having a given value, then x increases with μ, for if μ is increased the perpendicular distance between the two rays is increased; but if we increase a we must increase μ if $\mu \cos 2a$ is to remain a constant, so that μ should be as large as possible. The glass having been selected a should be chosen as large as is consistent with the proper value of θ. The angle a was taken 30°, making $\theta = 58°$ and x about 2.75, μ being 1.6936.

The definition of the reflected image will be affected by the dispersion of the light in entering the under surface of the plate. The magnitude of this effect is obviously calculable from the above equation. In the particular glass employed, both the plates and the prisms being of heavy flint glass, the indices of the refraction were $\mu_D = 1.6936$ and $\mu_E = 1.7032$. This gave for the values of x corresponding to the lines D and E the values 2.757 and 2.807. The variation in the value of x thus produced was about 1.8 per cent, corresponding to a difference in the value of d of about .001 mm., d being .05 mm. Thus the dispersion does not seriously injure the definition of the reflected image, and the latter, though somewhat weaker than the direct image, is of excellent definition.

The object of using two prisms G and H and thus an image reflected both in front of and behind the fiducial line is to correct for any error in horizontality of the plate E, and the axis of the observing telescope. In order to examine the effect of this let β be the angle between the axis of the telescope and the horizontal, and let γ be the angle between the horizontal and the intersection of the surface of the plate with a vertical plane parallel to the plane of the figure. The values of x and x_1 are given by the formulae:

$$x = \frac{2 \sin [2a + (\gamma - \beta)] \tan [\sin^{-1} \{\mu \cos 2a + (\gamma - \beta)\} + \beta]}{\cos \beta + \sin \beta \tan [\sin^{-1} \{\mu \cos 2a + (\gamma - \beta)\} + \beta]}$$

$$x_1 = \frac{2 \sin [2a - (\gamma - \beta)] \tan [\sin^{-1} \{\mu \cos 2a - (\gamma - \beta)\} - \beta]}{\cos \beta - \sin \beta \tan [\sin^{-1} \{\mu \cos 2a - (\gamma - \beta)\} - \beta]}$$

It is immediately evident, that small variations from the horizontal, changing x in one direction, will change x_1 to about the same extent in the opposite direction, so that the mean magnifying power of the two

prisms is unchanged by small values of β and γ. This can be illustrated by the following table of values of x and x_1, and the errors introduced when the mean is taken.

			x	x_1	Mean.	Error.
$\alpha = 30°$	$\mu_b = 1.6936$	$\beta = \gamma = 0$	2.7574	2.7574	2.7574	.000%
" = "	" = "	$\beta = 10', \gamma = 0$	2.8073	2.7088	2.7580	.025%
" = "	" = "	$\beta = 0, \gamma = 10'$	2.7136	2.8027	2.7581	.025%

If $d = .2$ mm. and $\beta = \gamma = 10'$ the error in using the mean of x and x_1 is 0.0001 mm.

If the plate is out of the horizontal by having been rotated around a horizontal axis parallel to the scales, the reading of one edge line will be increased and the other decreased to the same extent, so that the error is eliminated in the mean.

The light coming by reflection from the rear prism H has a somewhat longer path than the light coming from the front prism; therefore to have all the images in focus at once, a block of glass of requisite length, in this case 2.1 cm., is inserted in the path of the light from the rear prism. It is not shown in the drawing.

The advantages of the above procedure may be summed up as follows: The images observed cannot be displaced and distorted, as the path of the light lies entirely in air and optically worked glass. All errors of levelling are eliminated. When used in a constant-volume gas thermometer the quantity of gas between the lower meniscus and the thermometer bulb is reduced very much. The breadth of the beam of light coming to the telescope is unrestricted, so that the resolving power is not cut down. The meniscus does not have to be brought to a definite point, but only near it, the remaining distance then being read in the cathetometer telescope. This is especially convenient in barometer work where two surfaces are to be simultaneously read, and the setting of one alters the other.

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